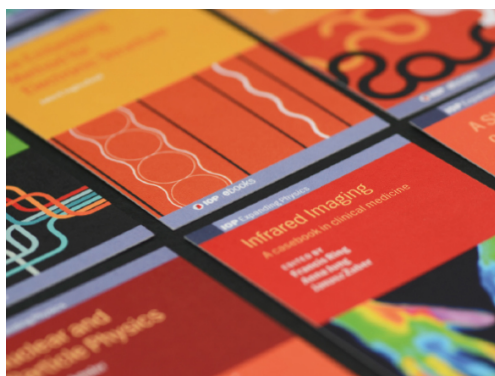


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The Cosmological Parameter: Constant or Dynamical

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Abstract. After providing a brief history of the cosmological constant, some early cosmological models are discussed. The most popular current cosmological model is the Λ CDM model, but there are several problems associated with this model, e.g., the cosmological constant problem and fine tuning. The idea of a dynamical cosmological parameter (Λ) is then introduced, which appears to fit observations better, and which can solve many problems associated with a constant Λ . The relationship between a variable Λ and bulk viscosity is then elucidated. It is then shown that some versions of $f(R, T)$ theory can be regarded as equivalent to general relativity with a variable Λ . Some Chaplygin gas cosmological models can also be re-cast as variable Λ models.

1. Introduction

The cosmological constant or parameter, Λ , has a long and interesting history. It was first introduced by Einstein in 1917 [1] in order to obtain a static model of the universe. At that time, it was believed that the universe was static, and the evidence of an expanding universe came later with the observations of Slipher and Hubble [2,3]. Thereafter Einstein abandoned the cosmological constant, calling it the biggest blunder of his life. However, as the saying goes, once a genie is let out of the bottle, it is very difficult to get it back in again. In 1988 [4-6], the discovery of an expanding universe has revived interest in the term, and it plays a very important role in explaining dark energy. The so-called Λ CDM model is the most widely accepted explanation for dark energy [7]. There are several problems associated with the cosmological constant, such as the cosmological constant problem [8]. Current observations indicate that the value of Λ is fairly small, whilst particle physics predicts a much larger value by 120 orders of magnitude. This remains a major outstanding problem in theoretical cosmology today.

There are now several reasons to believe in a dynamical cosmological parameter. It appears to offer a better fit to observations [9], and can solve the cosmological constant problem [10]. A dynamical Λ also arises naturally in some theories, such as scalar-tensor theories [11] and $f(R, T)$ theory [12].

2. Early history

Einstein's field equations in suitable units are:

$$R_{ab} - \frac{1}{2}g_{ab} = T_{ab} \quad (1)$$



He found that these equations did not admit a static solution. So he introduced a cosmological constant Λ into his field equations:

$$R_{ab} - \frac{1}{2}Rg_{ab} + \Lambda g_{ab} = T_{ab} \quad (2)$$

Using the above equations, Einstein was able to get a static model, called the Einstein static universe [1]. The Friedmann-Lemaitre-Robertson-Walker (FLRW) metric is:

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right] \quad (3)$$

where a is the scale factor (or radius), $k = 0, \pm 1$, and (r, θ, φ) are usual spherical coordinates. The parameters of this model are:

$$a = \text{const}, \quad \Lambda = \text{const}, \quad \mu = \text{const}, \quad p = \text{const}, \quad k = +1 \quad (4)$$

where μ and p are the energy density and pressure, respectively. In his derivation, Einstein tried to incorporate Mach's principle, which states that matter is necessary for a non-flat geometry.

Several interesting developments then followed. In 1917 [13], the Dutch mathematician Willem De Sitter found an empty expanding solution with nonzero Λ :

$$\Lambda = \text{const}, \quad a \approx \exp\left(\sqrt{\Lambda/3}t\right), \quad \mu = 0 = p, \quad H = \sqrt{\Lambda/3}, \quad k = 0 \quad (5)$$

This solution has been re-incarnated at least thrice:

- The steady state model of Bondi, Gold and Hoyle [14]
- Inflation ($\mu = -p \geq 0$)
- Current acceleration due to dark energy – Λ CDM model

From about 1910 to the mid 1920's, Slipher had discovered the redshift of several galaxies. Then in 1922, The Russian Friedmann found nonstatic models without the cosmological constant, the first person to mathematically predict an expanding universe [15]. The Belgian cleric Lemaitre appears to have independently derived the Friedmann solutions in 1927 [16]. During the period 1920-1930's, Robertson and Walker also independently derived the homogeneous and isotropic solutions, hence the term FLRW. The idea of an expanding universe was not very popular at that time, and even Einstein initially dismissed Friedmann's work as a mathematical curiosity.

The history around that time is not very clear. In 1923, there is a postcard from Einstein in which he discards the cosmological constant. However, at the famous 1927 Solvay conference, Einstein opposed the expanding universe. It was only after Hubble's discovery in 1929 [3] that most scientists accepted the idea of an expanding universe. According to the recollections of Gamow, Einstein called the introduction of the cosmological constant the biggest blunder of his life. By the 1950's, Λ had been generally rejected. It re-surfaces from time to time, sometimes for the wrong reasons, e.g., to reconcile the age of the universe. However, as the saying goes, once a genie is let out of the bottle, it is very difficult to get it back in again.

3. Dynamical cosmological parameter

3.1. Simple Variable Lambda

There are many theories or formalisms that allow for a dynamical cosmological parameter. The simplest one is simply to move the Λ term in equation (2) to the right side:

$$R_{ab} - \frac{1}{2}Rg_{ab} = \Lambda g_{ab} + T_{ab} = \bar{T}_{ab} \quad (6)$$

and then to allow Λ to vary, incorporating it into the energy-momentum tensor. This enables several problems associated with the usual cosmological model to be solved, e.g., the singularity, horizon, monopole and the cosmological constant problem [17] and can provide a better fit to observations [9].

3.2. Variable Lambda and G

If we write equation (2) specifically with Newton's gravitational constant G , then we get:

$$R_{ab} - \frac{1}{2}Rg_{ab} + \Lambda g_{ab} = GT_{ab}$$

If we then allow both Λ and G to vary, the Bianchi identities then yield:

$$\Lambda_{,b}g^{ab} = G_{,b}T^{ab} + T^{ab}_{;b}$$

Hence, we can still have the “usual” energy conservation law $T^{ab}_{;b} = 0$ holding if

$$\Lambda_{,b}g^{ab} = G_{,b}T^{ab}$$

This means that a variation of G is compensated for by a variation of Λ . However, note that in this formalism, a decreasing Λ requires an increasing G [18].

3.3. Relationship to viscosity

For the $k = 0$ FLRW metric, equation (2) yields:

$$3H^2 = \rho + \Lambda \quad (7)$$

$$\dot{\rho} + 3H(\rho + p) = -\dot{\Lambda} \quad (8)$$

To compare with bulk viscous models, we consider the Friedmann and conservation equations as obtained from equation (1):

$$3H^2 = \mu \quad (9)$$

$$\dot{\mu} + 3H(\mu + p) = 0 \quad (10)$$

where we have used μ for reasons that will become clear later. To obtain the equations for bulk viscosity, we replace p by $p - 3\eta H$:

$$p' = p - 3\eta H \quad (11)$$

where η is the coefficient of bulk viscosity.

Substituting equation (11) into equation (10) yields:

$$\dot{\mu} + 3H(\mu + p) = 9\eta H^2 \quad (12)$$

where we have dropped the prime for convenience. To compare equations (7) and (8) with equations (9) and (10), we now let

$$\mu = \rho + \Lambda \quad (13)$$

Equations (7) and (8) then become

$$3H^2 = \mu \quad (14)$$

$$\dot{\mu} + 3H(\mu + p) = 3H\Lambda \quad (15)$$

From equations (12) and (15), we then see that variable Lambda and bulk viscous models are identical if [19]

$$\Lambda = 3\eta H \quad (16)$$

3.4. $f(R, T)$ Theory

The action for $f(R, T)$ theory is:

$$S = \frac{1}{16\pi} \int f(R, t) \sqrt{-g} d^4x + \int L_m \sqrt{-g} d^4x \quad (17)$$

where $f(R, T)$ is some function of the Ricci scalar R and the trace T of the energy-momentum tensor. Varying the action (17), we get:

$$f_R(R, T)R_{ab} - \frac{1}{2}f(R, T)g_{ab} + (g_{ab}\square - \nabla_a \nabla_b)f_R(R, T) = T_{ab} - f_T(R, T)T_{ab} - f_T(R, T)\Theta_{ab} \quad (18)$$

where $\square = \nabla_a \nabla_b$, $f_R(R, T) = \frac{\partial f(R, T)}{\partial R}$, $f_T = \frac{\partial f(R, T)}{\partial T}$ and ∇_a denotes the covariant derivative. The expansion tensor Θ_{ab} is given by:

$$\Theta_{ab} = -2T_{ab} + g_{ab}L_m - 2g^{cd} \frac{\partial^2 L_m}{\partial g^{ab} \partial g^{cd}} \quad (19)$$

Consider a perfect fluid:

$$T_{ab} = (p + \rho)u_a u_b + pg_{ab} \quad (20)$$

and choose the matter Lagrangian as $L_m = -p$. Then equation (20) becomes:

$$\Theta_{ab} = -2T_{ab} - pg_{ab} \quad (21)$$

We then choose $f(R, T)$ as:

$$f(R, T) = R + 2\lambda T \quad (22)$$

where λ is a constant. The field equations (18) then become

$$R_{ab} - \frac{1}{2}Rg_{ab} = (1 + 2\lambda)T_{ab} + (\rho - p)\lambda g_{ab} \quad (23)$$

Let us recall Einstein's equation with cosmological constant on the right side,

$$R_{ab} - \frac{1}{2}Rg_{ab} = T_{ab} - \Lambda g_{ab} \quad (24)$$

By comparing the above two equations, and taking the coupling parameter λ to be small, we see that an effective cosmological parameter as a function of T may be defined in $f(R, T)$ as:

$$\Lambda = \Lambda(T) = -(2p + T)\lambda = (p - \rho)\lambda \quad (25)$$

Thus, we can also regard this form of $f(R, T)$ theory [$f(R, T) = R + 2\lambda T$] for a perfect fluid as equivalent to general relativity with an effective cosmological parameter [20,21].

4. Conclusion

The cosmological constant Lambda was introduced by Einstein to get a static model of the universe. It was rejected when the expansion of the universe was discovered. Now, it is necessary for inflation and late time acceleration. A dynamic Lambda can solve many problems: cosmological constant problem, initial singularity, monopole problem, flatness problem. It fits observations better than a constant Lambda. There is a close relationship to bulk viscous models, the scale covariant theory, certain versions of $f(R, T)$ and Brans-Dicke theories, and Chaplygin gas models.

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